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# SOME RELATIONS CONNECTING ANDREW'S MOCK THETA FUNCTIONS

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## ABSTRACT

In his last letter to Hardy, Ramanujan gave a list of seventeen functions F(q), where q is a complex number and |q|<1 and called them "mock theta functions". He called them mock theta functions as they were not theta functions. He further stated that as q radially approaches to any point $e^{2\pi ir}$ ('r' rational) there is a theta function $\theta_r(q)$ such that $F(q)-\theta_r(q)=0(1)$.Moreover there is no single theta function which works for all r i.e. for every theta function $\theta(q)$ ,there is some root of unity "r" for which $F(q)-\theta(q)$ is unbounded as $q\to e^{2\pi ir}$ radially. In this paper we obtain relations connecting mock theta functions, partial mock theta functions of Andrews [2] and infinite products analogous to the identities of Ramanujan.

**KEYWORDS:** Mock Theta Functions, Partial Mock Theta Functions

## 1. INTRODUCTION

Ramanujan's last mathematical creation was his mock theta function which he discovered during the last years of his life. Ramanujan gave a list of seventeen mock theta functions and labelled them as third, fifth and seven orders without giving any reason for his classification.

A mock theta function is a function f(q) defined by a q-series, convergent for |q|<1 which satisfies the following two conditions :

- For every root of unity $\xi$ there is a $\theta$-function $\theta_\xi(q)$ such that the difference f(q)– $\theta_\xi(q)$ is bounded as $q\to\xi$ radially.
- There is no single $\theta$–function which works for all $\xi$ i.e. for every $\theta$–function $\theta(q)$ there is some root of unity $\xi$ for which f(q)– $\theta(q)$ is unbounded as $q\to\xi$ radially.

For mock theta function $\overline{\psi}_1(q)$,

$$\overline{\psi}_1(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(-q;q)_{2n}} \tag{1.1}$$

the partial mock theta function will be defined and denoted as

$$\overline{\psi}_{1,N}(q)=\sum_{n=0}^{N}\frac{q^{n^2}}{(-q;q)_{2n}}$$

(1.2)

The mock theta functions of Andrews [2] are

$$\overline{\psi}_0(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(-q;q)_{2n}} \tag{1.3}$$

$$\overline{\psi}_1(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2+2n}}{(-q;q)_{2n+1}} \tag{1.4}$$

$$\overline{\psi}_2(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2+2n}\left(q;q^2\right)_n}{\left(q^2;q^2\right)_n(-q;q)_{2n}} \tag{1.5}$$

$$\overline{\psi}_3(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}(-q;q)_n^2}{(q;q)_{2n}} \tag{1.6}$$

Partial mock theta functions of the above mock theta functions are

$$\overline{\psi}_{0,m}(q)=\sum_{n=0}^{m}\frac{q^{2n^2}}{(-q;q)_{2n}} \tag{1.7}$$

$$\overline{\psi}_{1,m}(q)=\sum_{n=0}^{m}\frac{q^{2n^2+2n}}{(-q;q)_{2n+1}} \tag{1.8}$$

$$\overline{\psi}_{2,m}(q)=\sum_{n=0}^{m}\frac{q^{2n^2+2n}\left(q;q^2\right)_n}{\left(q^2;q^2\right)_n(-q;q)_{2n}} \tag{1.9}$$

$$\overline{\psi}_{3,m}(q)=\sum_{n=0}^{m}\frac{q^{n^2}(-q;q)_n^2}{(-q;q)_{2n}} \tag{1.10}$$

The following q-notations have been used

For $|q^k|<1$,

$$\left(a;q^k\right)_n=\prod_{j=0}^{n-1}\left(1-aq^{kj}\right), n\geq 1$$

$$\left(a;q^k\right)_0=1$$

$$\left(a;q^k\right)_\infty=\prod_{j=0}^{\infty}\left(1-aq^{kj}\right)$$

$$(a)_n=(a;q)_n$$

$$\left(a_1,a_2\ldots,a_m;q^k\right)_n=\left(a_1;q^k\right)_n\left(a_2;q^k\right)_n\ldots\left(a_m;q^k\right)_n$$

Ramanujan, in chapter 16 of his second note book defined theta functions as follows;[18,6]:

$$\chi(q)=\sum_{n=0}^{\infty} q^{\frac{n(n+1)}{2}}=\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty} \tag{1.11}$$

An identity due to Euler is [9,chap 17]

$$\sum_{n=0}^{\infty}\frac{q^{(n+1)^2}}{(q;q^2)_{n+1}}=(-x;q)_\infty \tag{1.12}$$

The special cases of the above identity are

$$L(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q^2;q^2)_n}=\frac{(q^2,q^2,q^4;q^4)_\infty}{(q;q)_\infty} \tag{1.13}$$

$$T(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q^2;q^2)_n}=\frac{(q,q^3,q^4;q^4)_\infty}{(q;q)_\infty} \tag{1.14}$$

Jackson[15] discovered the following identity

$$U(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q;q)_{2n}}=\frac{(-q^3,-q^5,q^8;q^8)_\infty}{(q^2;q^2)_\infty} \tag{1.15}$$

This identity was independently discovered by Slater [19,(39)]who also discovered its companion identity

$$V(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q;q)_{2n+1}}=\frac{(-q,-q^7,q^8;q^8)_\infty}{(q^2;q^2)_\infty} \tag{1.16}$$

The famous Roger's- Ramanujan identities are

$$M(q)=\sum_{n=0}^{\infty}\frac{q^{n^2}}{(q;q)_n}=\frac{1}{(q,q^4;q^5)_\infty} \tag{1.17}$$

$$N(q)=\sum_{n=0}^{\infty}\frac{q^{n(n+1)}}{(q;q)_n}=\frac{1}{(q^2,q^3;q^5)_\infty} \tag{1.18}$$

The identity analogous to the Rogers-Ramanujan identity is the so-called Gollnitz-Gordon identity given by [10,11]

$$E(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n q^{n^2}}{(q^2;q^2)_n}=\frac{1}{(q;q^8)_\infty(q^4;q^8)_\infty(q^7;q^8)_\infty} \tag{1.19}$$

$$\eta(q)=\sum_{n=0}^{\infty}\frac{(-q;q^2)_n q^{n^2+2n}}{(q^2;q^2)_n}=\frac{1}{(q^3,q^4,q^5;q^8)_\infty} \tag{1.20}$$

Hahn defined the septic analogues of the Roger's-Ramanujan functions as [13,14]

$$X(q)=\sum_{n=0}^{\infty}\frac{q^{2n^2}}{(q^2;q^2)_n(-q;q)_{2n}}=\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.21}$$

$$Y(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n}}=\frac{(q^2,q^5,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.22}$$

$$Z(q)=\sum_{n=0}^{\infty}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n+1}}=\frac{(q,q^6,q^7;q^7)_\infty}{(q^2;q^2)_\infty} \tag{1.23}$$

The nonic analogues of Rogers-Ramanujan functions are [5,equations (1.6),(1.7),(1.8)]

$$O(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n}q^{3n^2}}{(q^3;q^3)_n(q^3;q^3)_{2n}}=\frac{(q^4,q^5,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.24}$$

$$Q(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n}(1-q^{3n+2})q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}}=\frac{(q^2,q^7,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.25}$$

$$W(q)=\sum_{n=0}^{\infty}\frac{(q;q)_{3n+1}q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}}=\frac{(q,q^8,q^9;q^9)_\infty}{(q^3;q^3)_\infty} \tag{1.26}$$

The partial sums of the above functions are given as follows

$$\chi_m(q)=\sum_{n=0}^{m}q^{\frac{n(n+1)}{2}} \tag{1.27}$$

$$L_m(q)=\sum_{n=0}^{m}\frac{q^{n^2}}{(q^2;q^2)_n} \tag{1.28}$$

$$T_m(q)=\sum_{n=0}^{m}\frac{q^{n(n+1)}}{(q^2;q^2)_n} \tag{1.29}$$

$$U_m(q)=\sum_{n=0}^{m}\frac{q^{2n^2}}{(q;q)_{2n}} \tag{1.30}$$

$$V_m(q)=\sum_{n=0}^{m}\frac{q^{2n(n+1)}}{(q;q)_{2n+1}} \tag{1.31}$$

$$M_m(q)=\sum_{n=0}^{m}\frac{q^{n^2}}{(q;q)_n} \tag{1.32}$$

$$N_m(q)=\sum_{n=0}^{m}\frac{q^{n(n+1)}}{(q;q)_n} \tag{1.33}$$

$$E_m(q)=\sum_{n=0}^{m}\frac{(-q;q^2)_n q^{n^2}}{(q^2;q^2)_n} \tag{1.34}$$

$$\eta_m(q)=\sum_{n=0}^{m}\frac{(-q;q^2)_n q^{n^2+2n}}{(q^2;q^2)_n} \tag{1.35}$$

$$X_m(q)=\sum_{n=0}^{m}\frac{q^{2n^2}}{(q^2;q^2)_n(-q;q)_{2n}} \tag{1.36}$$

$$Y_m(q)=\sum_{n=0}^{m}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n}} \tag{1.37}$$

$$Z_m(q)=\sum_{n=0}^{m}\frac{q^{2n(n+1)}}{(q^2;q^2)_n(-q;q)_{2n+1}} \tag{1.38}$$

$$O_m(q)=\sum_{n=0}^{m}\frac{(q;q)_{3n}q^{3n^2}}{(q^3;q^3)_n(q^3;q^3)_{2n}} \tag{1.39}$$

$$Q_m(q)=\sum_{n=0}^{m}\frac{(q;q)_{3n}(1-q^{3n+2})q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}} \tag{1.40}$$

$$W_m(q)=\sum_{n=0}^{m}\frac{(q;q)_{3n+1}q^{3n(n+1)}}{(q^3;q^3)_n(q^3;q^3)_{2n+1}} \tag{1.41}$$

## 2. METHODOLOGY

We shall make use of the following known identity of Srivastava [20]:

$$\sum_{m=0}^{\infty}\delta_m\sum_{r=0}^{m}\alpha_r=\left(\sum_{r=0}^{\infty}\alpha_r\right)\left(\sum_{m=0}^{\infty}\delta_m\right)-\sum_{r=0}^{\infty}\alpha_{r+1}\sum_{m=0}^{r}\delta_m \tag{2.1}$$

## 3. RESULTS

We shall make use of the known identity (2.1) to obtain relations connecting Andrews mock theta functions, partial mock theta functions and infinite products analogous to the identities of Ramanujan.

- Taking $\delta_m=q^{\frac{m(m+1)}{2}}$ in (2.1) and by (1.11) and (1.27) we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\sum_{r=0}^{\infty}\alpha_r=\sum_{m=0}^{\infty}q^{\frac{m(m+1)}{2}}\sum_{r=0}^{m}\alpha_r+\sum_{r=0}^{\infty}\alpha_{r+1}\chi_m(q) \tag{3.1}$$

- Taking $\alpha_r=\dfrac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.1) and making use of (1.3) and (1.7) we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\overline{\psi}_0(q)=\sum_{m=0}^{\infty}q^{\frac{m(m+1)}{2}}\overline{\psi}_{0,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}\chi_m(q) \tag{3.2}$$

- Taking $\alpha_r=\dfrac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.1) and making use of (1.4) and (1.8) we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\overline{\psi}_1(q)=\sum_{m=0}^{\infty}q^{\frac{m(m+1)}{2}}\overline{\psi}_{1,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}\chi_m(q) \tag{3.3}$$

- Taking $\alpha_r=\dfrac{q^{2r^2+2r}(q;q^2)_r}{(q^2;q^2)_r\,(-q;q)_{2r}}$ in (3.1) and making use of (1.5) and (1.9)we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\overline{\psi}_2(q)=\sum_{m=0}^{\infty}q^{\frac{m(m+1)}{2}}\overline{\psi}_{2,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}(q;q^2)_{r+1}}{(q^2;q^2)_{r+2}(-q;q)_{2r+2}}\chi_m(q) \tag{3.4}$$

- Taking $\alpha_r=\dfrac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.1) and making use of (1.6) and (1.10) we get

$$\frac{(q^2;q^2)_\infty}{(q;q^2)_\infty}\overline{\psi}_3(q)=\sum_{m=0}^{\infty}q^{\frac{m(m+1)}{2}}\overline{\psi}_{3,m}(q)+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)^2_{r+1}}{(q;q)_{2r+2}}\chi_m(q) \tag{3.5}$$

(B) Taking $\delta_m = \frac{q^{m^2}}{(q^2;q^2)_m}$ in (2.1) and by (1.13) and (1.28) we get

$$\frac{(q^2,q^2,q^4;q^4)_\infty}{(q;q)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m} \sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}L_m(q) \tag{3.6}$$

- Taking $\alpha_r = \frac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.6) and using (1.3) and (1.7) we get

$$\frac{(q^2,q^2,q^4;q^4)_\infty}{(q;q)_\infty}\overline{\psi}_0(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}\overline{\psi}_{0,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}L_m(q) \tag{3.7}$$

- Taking $\alpha_r = \frac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.6) and using (1.4) ,(1.8) we get

$$\frac{(q^2,q^2,q^4;q^4)_\infty}{(q;q)_\infty}\overline{\psi}_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}\overline{\psi}_{1,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}L_m(q) \tag{3.8}$$

- Taking $\alpha_r = \frac{q^{2r^2+2r}(q;q^2)_r}{(q^2;q^2)_r(-q;q)_{2r}}$ in (3.6) and using (1.4) ,(1.9) we get

$$\frac{(q^2,q^2,q^4;q^4)_\infty}{(q;q)_\infty}\overline{\psi}_1(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q^2;q^2)_m}\overline{\psi}_{2,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}(q;q^2)_{r+1}}{(q^2;q^2)_{r+1}(-q;q)_{2r+2}}L_m(q) \tag{3.9}$$

- Taking $\alpha_r = \frac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.6) and using (1.6) ,(1.10) we get

$$\frac{\left(q^2,q^2,q^4;q^4\right)_\infty}{(q;q)_\infty}\overline{\psi}_3(q)=\sum_{m=0}^{\infty}\frac{q^{m^2}}{\left(q^2;q^2\right)_m}\overline{\psi}_{3,m}(q)+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)^2_{r+1}}{(q;q)_{2r+2}}L_m(q) \tag{3.10}$$

- Taking $\delta_m=\dfrac{q^{m(m+1)}}{\left(q^2;q^2\right)_m}$ in (2.1) and by (1.14) and (1.29) we get

$$\frac{\left(q,q^3,q^4;q^4\right)_\infty}{(q;q)_\infty}\sum_{r=0}^{\infty}\alpha_r=\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{\left(q^2;q^2\right)_m}\sum_{r=0}^{m}\alpha_r+\sum_{r=0}^{\infty}\alpha_{r+1}T_m(q) \tag{3.11}$$

- Taking $\alpha_r=\dfrac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.11) and making use of (1.3) and (1.7) we arrive at

$$\frac{\left(q,q^3,q^4;q^4\right)_\infty}{(q;q)_\infty}\overline{\psi}_0(q)=\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{\left(q^2;q^2\right)_m}\overline{\psi}_{0,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}T_m(q) \tag{3.12}$$

- Taking $\alpha_r=\dfrac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.11) and making use (1.4) and (1.8)we arrive at

$$\frac{\left(q,q^3,q^4;q^4\right)_\infty}{(q;q)_\infty}\overline{\psi}_1(q)=\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{\left(q^2;q^2\right)_m}\overline{\psi}_{1,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}T_m(q) \tag{3.13}$$

- Taking $\alpha_r=\dfrac{q^{2r^2+2r}\left(q;q^2\right)_r}{\left(q^2;q^2\right)_r(-q;q)_{2r}}$ in (3.11) and making use (1.5) and (1.9)we arrive at

$$\frac{\left(q,q^3,q^4;q^4\right)_\infty}{(q;q)_\infty}\overline{\psi}_2(q)=\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{\left(q^2;q^2\right)_m}\overline{\psi}_{2,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{\left(q^2;q^2\right)_{r+1}(-q;q)_{2r+2}}T_m(q) \tag{3.14}$$

- Taking $\alpha_r=\dfrac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.11) and making use of (1.6) and (1.10)we arrive at

$$\frac{(q,q^3,q^4;q^4)_\infty}{(q;q)_\infty}\overline{\psi}_3(q)=\sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q^2;q^2)_m}\overline{\psi}_{3,m}(q)$$
$$+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)_{r+1}^2}{(q;q)_{2r+2}}T_m(q) \tag{3.15}$$

Taking $\delta_m=\frac{q^{m^2}}{(q;q)_m}$ in (2.1) and by (1.17) and (1.32) we get

$$\frac{1}{(q,q^4;q^5)_\infty}\sum_{r=0}^{\infty}\alpha_r=\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}$$
$$\sum_{r=0}^{m}\alpha_r+\sum_{r=0}^{\infty}\alpha_{r+1}M_m(q) \tag{3.16}$$

Taking $\alpha_r=\frac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.16) and Making use of (1.3) and (1.7) we get

$$\frac{1}{(q,q^4;q^5)_\infty}\overline{\psi}_0(q)=\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\overline{\psi}_{0,m}(q)$$
$$+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}M_m(q) \tag{3.17}$$

Taking $\alpha_r=\frac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.16) and by (1.4) and (1.8) we get

$$\frac{1}{(q,q^4;q^5)_\infty}\overline{\psi}_1(q)=\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\overline{\psi}_{1,m}(q)$$
$$+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}M_m(q) \tag{3.18}$$

Taking $\alpha_r=\frac{q^{2r^2+2r}(q;q^2)_r}{(q^2;q^2)_r(-q;q)_{2r}}$ in (3.16) and making use of (1.5) and (1.9) we get

$$\frac{1}{(q,q^4;q^5)_\infty}\overline{\psi}_2(q)=\sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\overline{\psi}_{2,m}(q)$$
$$+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}(q;q^2)_{r+1}}{(q;q^2)_{r+1}(-q;q)_{2r+2}}M_m(q) \tag{3.19}$$

Taking $\alpha_r = \frac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.16) and making use of (1.6) and (1.10) we get

$$\frac{1}{(q,q^4;q^5)_\infty}\overline{\psi}_3(q) = \sum_{m=0}^{\infty}\frac{q^{m^2}}{(q;q)_m}\overline{\psi}_{3,m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)^2{}_{r+1}}{(q;q)_{2r+2}}M_m(q) \tag{3.20}$$

Taking $\delta_m = \frac{q^{m(m+1)}}{(q;q)_m}$ in (2.1) and by (1.18) and (1.33) we get

$$\frac{1}{(q^2,q^3;q^5)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}N_m(q) \tag{3.21}$$

Taking $\alpha_r = \frac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.21) and making use (1.3) and (1.7) we get

$$\frac{1}{(q^2,q^3;q^5)_\infty}\overline{\psi}_0(q) = \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\overline{\psi}_{0,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}N_m(q) \tag{3.22}$$

Taking $\alpha_r = \frac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.21) and making use of (1.4) and (1.8) we get

$$\frac{1}{(q^2,q^3;q^5)_\infty}\overline{\psi}_1(q) = \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\overline{\psi}_{1,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}N_m(q) \tag{3.23}$$

Taking $\alpha_r = \frac{q^{2r^2+2r}(q;q^2)_r}{(q^2;q^2)_r(-q;q)_{2r}}$ in (3.21) and making use of (1.5) and (1.9) we get

$$\frac{1}{(q^2,q^3;q^5)_\infty}\overline{\psi}_2(q) = \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\overline{\psi}_{2,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}(q;q^2)_{r+1}}{(q^2;q^2)_{r+1}(q;q)_{2r+2}}N_m(q) \tag{3.24}$$

Taking $\alpha_r = \dfrac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.21) and making use of (1.6) and (1.10) we get

$$\frac{1}{\left(q^2,q^3;q^5\right)_\infty}\overline{\psi}_3(q) = \sum_{m=0}^{\infty}\frac{q^{m(m+1)}}{(q;q)_m}\overline{\psi}_{3,m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)^2_{r+1}}{(q;q)_{2r+2}}N_m(q) \tag{3.25}$$

Taking $\delta_m = \dfrac{q^{2m^2}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}$ in (2.1) and by (1.21) and (1.36) we get

$$\frac{\left(q^3,q^4,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\sum_{r=0}^{\infty}\alpha = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{\left(q^2;q^2\right)_m(-q;q)_{2m}} \sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}\chi_m(q) \tag{3.26}$$

Taking $\alpha_r = \dfrac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.26) and using (1.3) and (1.7) we get

$$\frac{\left(q^3,q^4,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}\overline{\psi}_{0,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}X_m(q) \tag{3.27}$$

Taking $\alpha_r = \dfrac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.26) and using (1.4) and (1.8) we get

$$\frac{\left(q^3,q^4,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}\overline{\psi}_{1,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}X_m(q) \tag{3.28}$$

Taking $\alpha_r = \dfrac{q^{2r^2+2r}\left(q;q^2\right)_r}{\left(q^2;q^2\right)_r(-q;q)_{2r}}$ in (3.26) and using (1.5) and (1.9) we get

$$\frac{\left(q^3,q^4,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_2(q) = \sum_{m=0}^{\infty}\frac{q^{2m^2}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}\overline{\psi}_{2,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}\left(q;q^2\right)_{r+1}}{\left(q^2;q^2\right)_{r+1}(-q;q)_{2r+2}}X_m(q) \tag{3.29}$$

Taking $\alpha_r = \dfrac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.26) and using (1.6) and (1.10) we get

$$\frac{(q^3,q^4,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\overline{\psi}_3(q) = \sum_{m=0}^{\infty} \frac{q^{2m^2}}{(q^2;q^2)_m(-q;q)_{2m}}\overline{\psi}_{3,m}(q) + \sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)^2{}_{r+1}}{(q;q)_{2r+2}}X_m(q) \tag{3.30}$$

(G)Taking $\delta_m = \dfrac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m}}$ in (2.1) and by (1.22) and (1.37)we get

$$\frac{(q^2,q^5,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\sum_{r=0}^{\infty}\alpha_r = \sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m}} \sum_{r=0}^{m}\alpha_r + \sum_{r=0}^{\infty}\alpha_{r+1}Y_m(q) \tag{3.31}$$

Taking $\alpha_r = \dfrac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.31) and using (1.3) and (1.7) we have

$$\frac{(q^2,q^5,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\overline{\psi}_0(q) = \sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m}}\overline{\psi}_{0,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}Y_m(q) \tag{3.32}$$

Taking $\alpha_r = \dfrac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.31) and by (1.4) and (1.8)we get

$$\frac{(q^2,q^5,q^7;q^7)_\infty}{(q^2;q^2)_\infty}\overline{\psi}_1(q) = \sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{(q^2;q^2)_m(-q;q)_{2m}}\overline{\psi}_{1,m}(q) + \sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}Y_m(q) \tag{3.33}$$

Taking $\alpha_r = \dfrac{q^{2r^2+2r}(q;q^2)_r}{(q^2;q^2)_r(-q;q)_{2r}}$ in (3.31) and using (1.5) and (1.9)we get

$$\frac{\left(q^2,q^5,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_2(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}\overline{\psi}_{2,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}\left(q;q^2\right)_{r+1}}{\left(q^2;q^2\right)_{r+1}(-q;q)_{2r+2}}Y_m(q) \tag{3.34}$$

Taking $\alpha_r=\frac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.31) and using (1.6) and (1.10)we get

$$\frac{\left(q^2,q^5,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_3(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m}}\overline{\psi}_{3,m}(q)+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)_{r+1}^2}{(q;q)_{2r+2}}Y_m(q) \tag{3.35}$$

Taking $\delta_m=\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}$ in (2.1) and using (1.23) and (1.38) we get

$$\frac{\left(q,q^6,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\sum_{r=0}^{\infty}\alpha_r=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}\sum_{r=0}^{m}\alpha_r+\sum_{r=0}^{\infty}\alpha_{r+1}Z_m(q) \tag{3.36}$$

Taking $\alpha_r=\frac{q^{2r^2}}{(-q;q)_{2r}}$ in (3.34) and using (1.3) and (1.7)we get

$$\frac{\left(q,q^6,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_0(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}\overline{\psi}_{0,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)^2}}{(-q;q)_{2r+2}}Z_m(q) \tag{3.37}$$

Taking $\alpha_r=\frac{q^{2r^2+2r}}{(-q;q)_{2r+1}}$ in (3.34) and using (1.4) and (1.8) we have

$$\frac{\left(q,q^6,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_1(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}\overline{\psi}_{1,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}}{(-q;q)_{2r+3}}Z_m(q) \tag{3.38}$$

Taking $\alpha_r=\frac{q^{2r^2+2r}\left(q;q^2\right)_r}{\left(q^2;q^2\right)_r(-q;q)_{2r}}$ in (3.34) and using (1.5) and (1.9)we have

$$\frac{\left(q,q^6,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_2(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}\overline{\psi}_{2,m}(q)+\sum_{r=0}^{\infty}\frac{q^{2(r+1)(r+2)}\left(q;q^2\right)_{r+1}}{\left(q^2;q^2\right)_{r+1}(-q;q)_{2r+2}}Z_m(q) \tag{3.39}$$

Taking $\alpha_r=\frac{q^{r^2}(-q;q)_r^2}{(q;q)_{2r}}$ in (3.34) and using (1.6) and (1.10) we have

$$\frac{\left(q,q^6,q^7;q^7\right)_\infty}{\left(q^2;q^2\right)_\infty}\overline{\psi}_3(q)=\sum_{m=0}^{\infty}\frac{q^{2m(m+1)}}{\left(q^2;q^2\right)_m(-q;q)_{2m+1}}\overline{\psi}_{3,m}(q)+\sum_{r=0}^{\infty}\frac{q^{(r+1)^2}(-q;q)_{r+1}^2}{(q;q)_{2r+2}}Z_m(q) \tag{3.40}$$

In the same way by assuming

$$\delta_m=\frac{\left(-q;q^2\right)_m q^{m^2}}{\left(q^2;q^2\right)_m},\frac{q^{m^2+2m}\left(-q;q^2\right)_m}{\left(q^2;q^2\right)_m},\frac{q^{2m^2}}{(q;q)_{2m}},\frac{q^{2m(m+1)}}{(q\ ;q\ )_{2m+1}},\frac{q^{3m^2}(q;q)_{3m}}{\left(q^3;q^3\right)_m\left(q^3;q^3\right)_{2m}},$$

$$\frac{(q;q)_{3m}\left(1-q^{3m+2}\right)q^{3m(m+1)}}{\left(q^3;q^3\right)_m\left(q^3;q^3\right)_{2m+1}} \text{ and } \frac{q^{3m(m+1)}(q;q)_{3m+1}}{\left(q^3;q^3\right)_m\left(q^3;q^3\right)_{2m+1}}$$

we can obtain relation connecting Andrews mock theta functions and the infinite products E(q) , $\eta(q)$, U(q), V(q),O(q),Q(q) and W(q) respectively.

## 4. CONCLUSIONS

In this paper relations connecting Andrews mock theta functions and its partial sums are obtained. Combinatorial interpretations of these results are under investigations and will be published later.

## REFERENCES


1. Agarwal R.P[1969]. Certain basic hypergeometric identities associated with mock theta functions. Quart. Jour.Math.20:121-128.
2. Andrews G.E. [2012]: q-orthogonal polynomials, Roger-Ramanujan Identities and mock theta functions; Proceeding of the steklov Institute of Mathematics Vol. 276, Issue 1;21-32
3. Andrews G.E. [1981] Ramanujan's lost notebook-1: Partial(-) functions.Adv. Math. 41:137-170
4. Andrews G.E. and Hickerson, D.[1991]: Ramanujan's lost notebook-III: The sixth order mock theta functions. Adv. Math, 89:60-105
5. Bailey, W.N.[1947]: Some identities in combinatory analysis. Proc. London Math. Soc. 49:421-435
6. Berndt, B.C.[1991]: Ramanujan's Notebooks - Part III. Springer, New York.
7. Choi Y.S [1991]: Tenth order mock theta functions in Ramanujan's lost notebook. Invent. Math.136: 497-596.

8. Denis, R.Y., Singh, S.N. and Singh, S.P. [2006] On certain relation connecting mock theta functions, Italian Jour. Pure and Appl. Math, 19:55-60.
9. Euler,L.[1748]:Introduction in Analysin Infinitorum, Marcum-Michaelem Bousuet, Lausanne
10. Gollinitz. H[1967]: Partition mit Differenzenbedingunger J. Reine Angew Math. 225:154-190.
11. Gordon, B.[1965]: Some continued fractions of Rogers-Ramanujan type. Duke Math. Jour.32:741-448
12. Gordon, B. and Mc Intosh, R.J. [2000]: Some eight order mock theta functions, jour, London Math soc.62: 321-365
13. Hahn,H.[2003]:Septic Analogues of the Rogers-Ramanujan functions Acta Arith, 110:381-399.
14. Hahn,H[2004]: Einstein series, analogues of the Rogers-Ramanujan functions and partition. Ph. D. Thesis University of Illinois at Urbana- Champaign
15. Jackson, F.H.[1928]:Examples of a generalization of Euler's transformation for power series. Messenger of Math 57:169-187
16. Mc. Intosh R. J. [2007]: Second order mock theta functions, Canad. Math. Bull 50(2):284-290
17. Ramanujan S. [1919]: Proof of certain identities in combinatory analysis Proc. comb. Philos. Soc. 19:214-216.
18. Ramanujan S [1957]: Ramanujan Notebooks (Vol I and Vol II), Tata Institute of Fundamental Research, Bombay.
19. Rogers L.J. [1894]: Second memoir on the expansion of certain infinite products proc. London Math. Soc. 25:3 18-343.
20. Slater L.J.[1952]: Further identities of the Rogers-Ramanujan type. Proc. London Math. Soc, 54(2):147-167.
21. Srivastava A. K. [1997]: On partial sums of mock theta functions of order three, proc. Indian Aca. Sci(Math. Sci) 107(1); 1-12.
22. Watson G. N. [1936]: The final problem: an account of the mock theta functions, Jour. London Math. Soc. 11: 55-80.